THE ROLE OF ANALOGIES AND ANCHORS IN REVISING MISCONCEPTIONS IN STATISTICS

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Though statistics is prevalent in everyday use, misconceptions about various concepts have been the object of decades of research. One way to overcome these false beliefs is through the use of anchoring situations, as advocated by Clement (1987) and Fast (1997). This paper uses a similar framework, with misconception-prone situations in Version A of the questionnaire, and their analogous anchoring counterparts in Version B. Results with 25 college social science students, coupled with in-depth interviews of 20 of them, anchoring situations could be effectively used to overcome them. Implications for pedagogy and research conclude the paper.

INTRODUCTION

The field of stochastics is both a pedagogical and research challenge. On one hand, the ideas of random sampling, correlations, probabilities, and the like abound in daily life. Not only mathematicians, but also laypeople use statistical terms in ways ranging from the most casual to the most precise. Statistics is indispensable in areas as diverse as economics, weather forecasting, sports analysis, stock predictions, and psychological experiments, among others.

Yet statistics in mathematical discourse is more rigorous than the fuzzy ideas found in everyday notions. The former follows a formal set of well--founded rules, whereas the latter may function more along the lines of intuition, common sense, and gut feel. Many problems exist in the latter case. A significant amount of research has revealed that people are prone to a host of misconceptions, and that they tend to utilize fallible heuristics rather than rigorous principles (Tversky & Kahneman, 1973, 1977; Borovnik & Peard, 1996). Though handy, these heuristics fail in numerous situations, giving rise to inconsistencies such as the so-called "gambler's fallacy."

Herein lies the challenge for educators and researchers. Given that statistical misconceptions are so pervasive, what methods can be utilized to counter these false beliefs and replace them with mathematically appropriate ones? "Thinking mathematically demands more. It presupposes that the learner has a more or less rich pool of intellectual tools at their disposal: algebraic notation, symbolism, and so on. These are precisely the intellectual tools available to a mathematician—and precisely those lacking in the naive learner" (Pratt, 1998, p. 2).

As for research, the reasons for these fuzzy notions have already been—and are still being—analyzed (Tversky & Kahneman, 1977; Konold, 1989). But overt methods to overcome such misconceptions are also needed, and here is where research should also be concentrated. Efforts in this direction include the use of examples for categorization (Quilici & Mayer, 1996), aids for subgoal learning (Catrambone, 1995), mental models (Hong & O'Neil, 1992), and rewording and analysis of anomalous information (Graesser & McMahen, 1993; Lee-Chua, 2001).

A promising tactic to deal with learning misconceptions is the use of anchors and analogical reasoning. Clement (1987) used this approach to help students overcome false beliefs in physics. In the field of probability, Fast (1997) adhered to Clement's framework and cited Cox and Mouw's (1992) earlier study which utilized cues.

But this line of inquiry is still novel. "The use of anchors and the analogies approach to overcoming probability misconceptions had not been attempted in previous research" (Fast, 1997, p. 326). As far as the researcher knows, neither has it been attempted in the area of statistics. Therefore, the goal of this paper is to use the anchoring and analogies framework to investigate students' conceptual understanding of various statistical situations, and the role (if any) of analogies in dealing with misconceptions that may arise.

SIGNIFICANCE OF THE STUDY

What makes the task of overcoming statistical misconceptions complex is that these erroneous ideas are highly resistant to change (Konold, 1989). One possible reason is that understanding and acquiring these concepts in the first place effort, and that changing them in the face of contradictory evidence is therefore resisted by the learner (Brown & Clement, 1987).

Overcoming misconceptions then requires change of concept construction—in cognitive psychological terms, a change of schema (Davis, 1984), In line with this goal, anchoring situations, which are conceptually isomorphic to misconception-prone situations, are generated. These anchors serve to draw out beliefs held by students which agree with formal statistical theory, and which therefore are expected to receive correct responses (Clement, 1987). The misconception-prone situations in this study have been deemed by researchers (e.g., Tversky & Kahneman, 1977) as often receiving wrong answers, which indicate mathematically incorrect concepts. Can the generated anchoring situations help students overcome their previously held statistics misconceptions?

Misconception-prone statistics situations are matched with conceptually isomorphic researcher-generated anchoring situations. The hypotheses to be tested in this paper are as follows:

1. Anchoring situations are more likely to result in more correct answers (thus, more correct concepts and schemas) rather than misconception-prone situations.
2. Anchoring situations can be used to help students overcome previously held situations misconceptions.

METHODOLOGY

Instruments

The procedures utilized in the present study use the framework advocated by Fast (1997), who credits Clement (1987) in turn. The topics studied in these three studies are different, though in the case of probability and statistics misconceptions, because of the intertwining nature of these two fields, there is necessarily some overlap. Clement (1987) worked in the area of physics misconceptions, while Fast (1997) studied probability misconceptions. In this paper, we look at statistics misconceptions.

Ten misconception-prone statistics situations were matched with conceptually isomorphic researcher-generated anchoring situations. The ten misconception-prone situations were compiled as Version A, and their ten analogous (anchoring) counterparts as Version B. Problems were given in multiple-choice format. Justifications for student answers were also requested.

Three samples questions (with both versions) are stated below, with reasons for their inclusion. The first case illustrates a misconception concerning the typicality effect.

Question 1 (Version A) A class of 50 students scored 60% on average in their statistics midterm examination. Three of them are your friends, and you discovered that two of the three got 90%. Can you estimate what score the third one received?

a. 60

b. 75

c. 90

Tversky and Kahneman (1977) assert that many individuals fell prey to the so-called typicality effect. They "mistake the most *typical* for the most *probable*" (Piatteli-Palmarini, 1994, p. 50). They are informed that the class mean is 60% but they disregard this information. Since they are also given the additional information that two people they know scored higher, they will be under the illusion that another friend scores equally high (90%). They will not heed the mathematically correct concept of mean, in which the correct answer of 60% is expected.

Let us look at the analogous (anchoring) counterpart of this question in Version B:

Question 1 (Version B) A class of 50 students scored 60% on average in their statistics midterm examination. Three of them are your friends, but two of the three would not tell you their scores. The third one likewise refused. Can you estimate what score the third one received?

a. 60

b. 75

c. 90

In this version, the only information given in the problem is that the class mean is 60%. Since no other data are provided, then the correct answer (60%) becomes apparent. The typicality heuristic cannot work in this case. Thus, this situation serves as an anchor for the satistically-correct concept that individual scores in a sample should not detract from consideration of the overall mean.

A second sample question revolves around the segregation fallacy, which is adopted from one of Tversky and Kahneman's (1977) classic experiments.

Question 2 (Version A) You just received a salary of P300,000 (P stands for Philippine pesos). Your boss gives you the chance to earn more. Which of the following scenarios will you choose?

a. You will receive P100,000 for certain.

b. You will toss a coin. If it turns up heads, you will get P200,000. If it turns up tails, you will get nothing at all.

c. It does not matter if you choose a or b.

It was found that most people will be conservative and choose the first scenario, that of receiving a certain sum for sure. In short, they fall prey to the so-called segregation fallacy, which means that when faced with a problem concerning choice, the "segregate their decision," that is, they accept it in the terms in which it is presented. They neglect to look for another alternative formulation. What is the mathematically correct answer? In this case, the statistical concept of expectation will eventually lead us to discover that there is no difference between the first and second scenarios—in the long run, we expect to receive a total of P400,000 (the original P300,000 plus the additional amount of P100,000).

Let us now look at the analogous (anchoring) counterpart in Version B.

Question 2 (Version B) You just received a salary of P300,000. Your boss gives you the chance to earn more. Which of the following scenarios will you choose?

a. You will receive P100,000 for certain.

b. You will receive the average of P200,000 and P0.

c. It does not matter if you choose a or b.

Framed in this way, the identical scenarios posed by both choices are readily apparent. The second statement is a direct (identical) alternative formulation of the first one, and the segregation effect should not be in play here. Piattelli-Palmarini puts it picturesquely, "[In the segregation effect], thanks to our cognitive sloth...we become prisoners of the frame we are offered" (1994, p. 57). But here the second scenario does not require much mental shift-work, so people should be more readily capable of shifting from one scenario to the other—rather than in the coin-tossing framework of Version A.

A third sample case focuses on the mistaken reliance on causality, which is different from the concept of correlation. Correlation is merely a statistical statement relating two properties of objects—there is no cause and effect involved.

Question 3 (Version A) The correlation between the colours of a mother's and daughter's eyes is 0.4. Which of the following situations has a higher chance of occurring?

a. The probability that a daughter's eyes are brown, given that her mother's eyes are brown.

b. The probability that a mother's eyes are brown, given that her daughter's eyes are brown.

c. There is no difference in the probabilities of a and b.

Research has shown that most people assign a higher probability to the first than the second (Weiner, 1985). But in a correlation, causality is never a factor, thus there should be no difference in probabilities between the two scenarios. The confusing aspect of all this is that people hark to the laws of genetics, where there is an asymmetrical relation between a parent's and a child's eye colours (e.g., the colour of a mother's eyes is genetically a link to that of her daughter's eyes, but not vice-versa). However, this genetic clue is not relevant to the mathematical notion of correlation. "Objective correlations of frequency do not reflect [the] asymmetrical relationship between cause and effect" (Piatteli-Palmarini, 1994, p. 79).

Let us now discuss its anchoring correlation counterpart in Version B.

Question 3 (Version B) The correlation between hair colour and intelligence has been found one controversial study to be 0.04. Which of the following situations has a higher chance of occurring?

a. The probability that someone is intelligent, given that he has black hair.

b. The probability that someone has black hair, given that he is intelligent.

c. There is no difference in the probabilities of a and b.

The sheer preposterousness of this situation (despite the phrase "controversial study" appended to it) readily leads people to the correct response, which is that there is no difference in the probabilities of the two cases. Since there are no popular biological or psychological findings to distract them from the proper notion of correlation, people tend to use the term correctly. They correctly surmise that correlation is a two-way (symmetrical) principle, and that what holds for one relation should also hold for the reverse.

SAMPLE

The subjects for this study were 25 sophomore social science majors from a university in Quezon City, Metro Manila, Philippines. All of them have taken an introductory statistics class one year before this research was conducted.

PROCEDURE

The subjects were first given the ten questions of Version A to complete. Their answers were then collected by the researcher—to prevent the possibility of returning to Version A after answering Version B. They were then immediately given Version B. No time limit was given, but all the subjects completed both versions of the questionnaire within one hour.

When the students submitted Version B, they were asked whether they could be interviewed. The purpose of the interview was to determine whether the anchors in Version B could be used to help students overcome their respective statistical misconceptions in Version A. All of the subjects agreed to be interviewed, but it was found later that five of them got perfect scores (on both versions), with appropriate written justifications for each question, thus they were excused from the interviews.

For the rest of the 20 subjects, each was individually interviewed by the researcher within two days of completing the questionnaire. During the interview, the students were presented with situations in Version A in which they had *wrong* answers (misconceptions) and the analogous counterparts (anchors) in Version B wherein they had *right* answers. In the interview, the researcher would guide them in an analogical reasoning process where hopefully the subjects would be led to correct their original incorrect answers in version A. Following Fast (1997), this change from a wrong to a right response would be interpreted as evidence of overcoming that particular statistical misconception.

The purpose of the interview was to examine whether statistical misconceptions could be dealt with by anchors. The results were categorized as either "Success," "Partial Success," or "Failure," coded 1, 0.5, or 0, respectively. "Success" means that during or after the interview, the subject changed his or her wrong response in Version A to the correct one, with the help of the analogical counterpart in Version B. "Partial Success" means that the subject changed his or her wrong response in Version A to the correct one, but without the help of the analogical counterpart in Version B (e.g., the change was due to other factors). It can also mean that the subject changed his or her wrong response in Version A to the correct one, but he or she was not convinced about the reasons for the change. "Failure" means that the subject did not change his or her incorrect response, or changed it to still another wrong one.

RESULTS AND DISCUSSION

Responses in Both Versions

In Version A (the misconception-prone version) 25 subjects answered 10 questions each, for a total of 250 answers. 138 were mathematically correct, thus, there were 112 wrong answers. In Version B (the anchoring version), out of 250 answers, 216 were correct, leaving 34 wrong responses. Each question in Version B resulted in more correct responses than its counterpart in Version A (see Table 1).

Table 1
Number of Correct Responses in Versions A and B

Question Number	Version A	Version B
1	19	24
2	13	23
3	12	21
4	16	22
5	13	21
6	18	25
7	11	20
8	11	20
9	13	20
10	12	20

Five subjects got perfect scores (all correct answers in both versions). For the rest of the 20 subjects, each student also obtained more correct responses in Version B than in Version A. These indicate that the situations in Version B rightfully served their purpose to encourage students to use correct schemas in understanding proper concepts. Based on the proportion of correct answers, therefore, mathematically correct schemas were used 138/250 or 55% of the time in Version A, compared to 216/250 or 86% in Version B. This difference is highly significant ($Z=7.6$, $p<0.01$).

Therefore, the first hypothesis of the study, that anchoring situations are more likely to result in more correct answers (thus, more correct concepts) rather than misconception-prone situations, had been supported.

In many situations, an incorrect answer in Version A was followed by a correct response in Version B. Of the 112 wrong answers in Version A, 100 were followed by correct answers in their counterparts in Version B, thus 89% of the questions answered incorrectly in Version A were answered correctly in the analogous Version B.

Subjects were also asked to justify their answers, and their replies supported the observation that the misconceptions (in Version A) were due to mathematically-incorrect schemas, due to the various fallacies described earlier (e.g., Tversky & Kahneman's research in 1977).

For instance, in response to why he chose the wrong answer "90%" for Question 1 in Version A, a subject wrote, "Because I know two other people who got 90%, so I reasoned that the other one should also get the same score." This statement represents the typicality effect. For Question 2 in Version A, to explain why she chose the first scenario (a sure P100,000), a subject replied, "Because I do not want to gamble. I want to be sure that I will get something." Her conservative stance indicates a tendency to use the segregation effect, to the detriment of considering alternative formulations. For Question 3 in Version A, as a justification for her wrong answer (she chose "a"), another subject explained, "I got my brown eyes from my mother, so I know 'a' is correct." Here we see the laws of genetics interfering with the appropriate mathematical notion of correlation.

Students were also asked to justify their answers for the questions in Version B, and their replies supported the observation that the anchoring effects of these questions led most of them to the right answer.

For instance, to justify her correct response for Question 1 in Version B, a subject replied, "The class average is 60%. Of course my best guess is that the third student got 60%." This explanation is the interpretation of the statistical notion of arithmetic mean. To explain his correct answer to Question 2 in Version B, a subject wrote, "There is no difference between the first and second scenarios. The first is just a shorter translation of the second, or put another way, the second is just a longer restatement of the first." Here the subject was able to think of a concept in more than one way, thus, he did not fall into the segregation fallacy. For her concept response to Question 3 in Version B, a subject reasoned, "Just because one thing is correlated to another, neither of them is a cause of the other." Clearly this implies that the statistical concept of correlation has been understood by the subject.

ANCHORING SITUATIONS

The 20 subjects who did not get perfect scores were interviewed, pinpointing those situations where possible anchors in Version B appeared for the misconceptions in Version A. As we have seen, out of the 112 wrong answers in Version A, 100 were followed by correct responses in Version B, thus there would appear to be 100 possible anchors. (The number of possible anchors per subject ranged from 1 to 7. See Table 2).

Based on the coding method presented previously, the points that best describe each subject's subsequent decision (whether to change the wrong response into the correct one—based on the analogy in Version B or based on a different reason altogether, to change the wrong response into another wrong one, or to retain the original wrong response) were computed. The proportion of these points to the total possible number of anchors was defined as the Success Rate. These results are shown in Table 2.

From Table 2, it can be seen from the interview results that in general, the analogies in Version B served as successful anchors to help students overcome their statistical misconceptions in Version A. Success rates ranged from 0.70 to 1.00, with 11 subjects having a 1.00 success rate. Thus, the hypothesis that anchoring situations can be used to help students overcome previously held statistics misconceptions has been supported.

Table 2
Results of Interviews with 20 Students

Student	Total Possible Anchors	Resulting Points After Interview	Success Rate
1	7	6	0.85
2	5	5	1.00
3	2	2	1.00
4	7	5.5	0.78
5	5	5	1.00
6	4	4	1.00
7	4	4	1.00
8	5	3.5	0.70
9	1	1	1.00
10	6	5.5	0.92
11	7	7	1.00
12	7	7	1.00
13	4	4	1.00
14	3	2.5	0.83
15	5	4	0.80
16	7	6	0.86
17	7	5	0.71
18	4	4	1.00
19	6	6	1.00
20	4	3	0.75

SUCCESSFUL ANCHORS

Following are selected transcripts from the interview sessions. (R stands for researcher, MS for male subject, and FS for female subject).

The first transcript is an example of an anchoring situation which was coded "Success."

R: *Here is Version A. Let's look at Question 2 again. Will you read it please?*

FS: *(reads the question)*

R: *You chose "a" yesterday. Will you stick with this answer?*

FS: *Yes.*

R: *Why?*

FS: *Because I don't like to gamble. I choose the first choice, because I am sure to get something, even if it is not the maximum amount.*

R: *Now here is Version B. Let's look at Question No. 2 here. Read it please.*

FS: *(reads the question)*

R: *You chose "c" yesterday. Do you still agree with this?*

FS: *Of course. This [sic] is so obvious. Letters "a" and "b" mean the same thing.*

R: *Now look at these questions again carefully. Do you see similarities between them?*

FS: *(studies for a while) Yes! I think the answer to Question 2 of this version (waves Version A) should be also "c."*

R: *Can you explain why?*

FS: *This tossing a coin business...there is a one-half chance that the coin will turn up heads, which means there is a one-half chance that I will get P200,000. There is also a one-half chance that tail will come up, so there is a one-half chance that I will get nothing. But when you add them up, it means the same thing...*

R: *The same thing as what?*

FS: *The same thing as getting P100,000. I think we studied something like this before. Our teacher said that the two are the same things. I mean, we should expect to get the same thing.*

R: *In the long run. Do you remember what this is called? I will give you a clue—you used the word "expect."*

FS: *I remember now. Mathematical expectation, sort of an average in the long run.*

Clearly, in this situation, the analogous question in Version B helped the subject change her wrong answer in Version A and justify it well. Most of the interviews ended along the same lines, as we have seen in Table 2, with most anchors becoming successful at helping students deal with previous misconceptions.

What accounts for the high success rates? Let us return to the notion of changing knowledge construction (revising schemas). According to constructivism, students come to thoroughly comprehend concepts only through their personal construction of knowledge (Davis, 1984). This "genuine" understanding is contrasted to mere "instrumental" knowledge, where the student knows how to go about particular routines but without thorough understanding of the concepts involved— and without learning how to apply them to new situations.

Anchors become useful then in revising schemas. Fast (1997, p. 327) explains, "Since misconceptions...have been acquired through constructivist activity, it is reasonable to apply the constructivist approach in the concept reconstruction process with the goal of acquiring mathematically correct concepts, or schemata." When situations where students are encouraged to answer correctly are generated, then anchors are established, and revision of schemas can proceed. These anchors can act as bridges to span what the students know to be correct to what they have previously falsely believed (Fast, 1997) or as scaffolds on which they are able to build a more rigorous understanding (Glaser, 1991).

PARTIALLY SUCCESSFUL AND UNSUCCESSFUL ANCHORS

The majority of the anchors in this study have been successful, as we have noted. However, some interviews were coded only "Partial Success," like the following:

R: *Here is a Version A. Let's look at Question No. 3 again. Will you read it please?*

MS: *(reads the question)*

R: *You chose "a" yesterday. Will you stick with this answer?*

MS: *I think so.*

R: *Why?*

MS: *In biology class the teacher was teaching us about genetics, and I think eye colour is inherited from generations [sic].*

R: *But look at it from the statistical point of view. Mathematically, 0.40 is just a correlation. Will you still stick with your answer?*

MS: *(shrugs, does not answer)*

R: *Now look at Version B. Read Question No. 3.*

MS: *(reads the question)*

R: *You chose "c" yesterday. Do you stick with this answer?*

MS: *Yes. There should be no relationship between intelligence and hair colour...Oh, well, actually, there is a relationship, but it is just a correlational one.*

R: *So what if it is a correlation?*

MS: *If it is a correlation, then we know that there is no cause and effect. At least that's what we were told last year. So the probability of the hair colour being this way for a person with a certain IQ should be the same as the probability of the vice-versa case [sic].*

R: *Now look at these two questions again carefully. Do you see any similarities between them?*

MS: *Well, both have correlations. The first version makes more sense than the second one. Wait, let me see here...*

MS: *(after a few minutes): Are you saying that the correct answer for the first question is that there is no difference between the two probabilities?*

R: *What do you think?*

MS: *Since you told me to compare the two questions, I suppose the answer should be "c."*

R: *But you are not convinced?*

MS: *Well, since this is also a correlation...I get your point. But I am really still not too sure of the whole thing. In math, maybe then the answer should be"c." But I am not too sure about biology. Does biology also use laws of statistics?*

R: *Yes, it does.*

MS: *Okay. So I change my answer, but I am still not sure.*

Here the subject was able to bring himself to consider the correct answer, with a little help from the analogous situation. But the anchor was not convincing enough in this case. The laws of genetics interfered too much with the statistical concept of correlation, so the subject was not convinced of the change. (However, we reiterate that in most of the other subjects, this anchor worked).

Let us now look at the third sample transcript, one of the few times when an anchor failed (coded "Failure"):

R: *Here is a Version A. Let's look at Question No. 1 again. Will you read it please?*

FS: *(reads the question)*

R: *You chose 90% yesterday. Would you stick with this answer?*

FS: *Yes, I would.*

R: *Why?*

FS: *Since two people I know got a high mark, then the next one will also get the same score.*

R: *Now here is Version B. Please read Question No. 1.*

FS: *(reads the question)*

R: *You chose 60% yesterday. Will you stick with this answer?*

FS: *Yes, I will.*

R: *Why?*

FS: *This is easy. The question says that the average of the class is 60%, so most probably, the person will get 60%, or at least near 60%.*

R: *Now look at the two questions again. Do you see any similarities between them?*

FS: *The two questions are all about finding the score of a third student. Also the class average in the two problems is the same. But there is a big difference. In Version A you know that two of the students got 90%. In Version B, you don't know anything.*

R: *Does this matter, if the class average is given?*

FS: I think it does. An average is just an average. You use it if you don't know anything more about the students. But once you know that two of them got 90%, then since you know the two, then it is the case that the third person [sic], whom you also know, will also get a similar grade.

R: But the class average is still 60%.

FS: Then that means that the other students, not these three friends of mine, are the ones who made the average go down.

In this case, the anchor failed. The subject was blinded by the typicality effect so that she disregarded the proper statistical meaning of average, even when it was pointed out to her. Again we turn to constructivism. In establishing schemata, students have to exert considerable time and effort, which make previously acquired concepts difficult to eradicate. Indeed erroneous ideas are difficult to erase, especially in personalized contexts (Myers, Hanse, Robson & McCann, 1983).

Another reason that some anchors are only partially successfully or totally unsuccessful is the competition among differing schemas. In some situations, a student may hold "parallel but different" schemas (Fast, 1997, p. 328)—one which is appropriate for dealing with the problem, and the other an incorrect one. Which one the student chooses depends on many factors, such as the representation of the problem, the wording of the situation, the context, the degree of abstraction, and so on (Perkins & Salomon, 1989). However, a more detailed examination of these various causes is beyond the scopes of this paper.

CONCLUSIONS AND RECOMMENDATIONS

This study aims to investigate college students' conceptual understanding of various statistical situations, and whether analogies can help them deal with misconceptions that may arise. The researcher-generated analogous situations served as anchors. As we have noted, these anchors drew out beliefs from students— beliefs which coincided with formal statistics and which therefore resulted in a majority of correct responses. At the same time, as shown by interview results, these anchors also succeeded in their role in guiding students to overcome previously held erroneous ideas. (However, it should be noted that not all anchors worked equally well for all students—the success rates ranged from 75% to 100%. It might well be that students who still held onto their wrong beliefs even after encountering analogies needed to hone other skills—emphasis on fundamental concepts, perhaps, or even strategies to dispel math anxieties).

What led students to possess misconceptions in the first place? The causes are myriad, and have been analyzed in detail by several researchers, the most notable being Tversky and Kahneman (1973, 1977). These fallacies in turn can be triggered by certain cues, such as the personalized context included in Question No. 1 in Version A, where friends of the respondent were falsely thought to play a major role in answering a query on arithmetic means. When this personalized trigger was removed in Version B, most subjects were able to reply correctly, and subsequently, used the latter as an anchor to revise their conceptions of the beliefs that appeared in Version A.

The choice of extreme situations, which in the case of Question 3, (Version A)) bordered on the absurd, also acted as an effective anchor. Where a relationship between two entities is deemed to be reasonable (whether the relation is causal or merely correlational), statistics is glossed over, and human intuition, at times highly fallible, takes over. But when a relationship strains common notions of reality, then subjects have no choice but to rely on mathematical concepts. Structuring both realistic and unrealistic correlations would be apt way to hone students' critical thinking skills.

How can the findings of this paper be applied in the classroom? Implications of this study for pedagogy include:

1. Be aware of the various misconceptions people have been shown to possess, and the reasons why they hold onto them.
2. Use extreme cases, de-personalized contexts, and the like to draw out these misconceptions.
3. Use analogies to serve as anchors for misconception-prone situations.
4. Ask students for their personal anchors, and discuss their effectiveness in various statistical situations.

Using analogies to overcome mathematical misconceptions is a fruitful field for research today. The causes for such misconceptions have already been analyzed (Tversky & Kahneman, 1977). Now the burden shifts from understanding the reasons for these false beliefs to ways of dealing with them, and analogical anchoring is one of the posited methods.

Recommendations for further research include:

1. Focus on analyzing which statistical situations serve as the most effective anchors, and the reasons behind their effectiveness.
2. In a similar vein, hypothesize certain situations where anchors may fail, and test these hypotheses. For instance, anchors may not succeed in situations where questions are framed ambiguously.
3. Study the factors which make students more readily accept anchors in reframing their former erroneous ideas. Such factors may include an ability to see connections between different topics, openness of mind, and an interest in mathematics in general.
4. Use the methodology discussed in this paper to deal with false beliefs in other fields of mathematics, such as calculus and geometry.

Statistics is indispensable in our lives. It is the task of educators and researchers alike to help students overcome statistical misconceptions, develop mathematically rigorous concepts, and learn to apply them to situations they have already encountered (and will likely face in the future).

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ACKNOWLEDGMENTS

The author wishes to acknowledge the professional chair given by the Ateneo de Manila University for SY 2002-2003.